Q1-

Use the substitution method to show that the recurrence

T(n) = √ n T( √ n) + n

has solution T(n) = O(n lg lg n).

Solution :

T(n) = √ n T( √ n) + n ----------------------------------- i

Let n = 2^m so √n = 2^m/2 m = logn

Now Substitute,

T(2^m) = 2^m/2 T(2^m/2) + 2^m ---------------------------- ii

Dividing by 2^m on both sides,

T(2^m) / 2^m = (2^m/2 T(2^m/2))/2^m + 2^m/ 2^m

T(2^m) / 2^m = (2^m/2 T(2^m/2))/2^m + 1 -------------------------- iii

Assume S(m) = T(2^m)/2^m

Substitute these Values in equation iii,

S(m) = S(m/2)+1 ------------------------------------------------------------------------ iv

Apply the Master theorem, T(n) = aT(n/b)+ f(n) where a = 1, b = 2, f(m) = 1

Calculate k = logb^a = log2^1 = 0

Compare n^k and f(m)

Condition of Master theorem,

1. n^k f(n) = O(f(n))
2. n^k f(n) = O(n^k)
3. n^k = f(n) = O(n^k logn)

m^0 = 1

1 = 1 it holds under rule-3

S(m) – m^0 log m put value of S(m) equation (iv)

T(2^m) / 2^m = log m

T(2^m) = 2^m log m, put these values in Eq(ii)

T(n) = O(n log log n) Prooved